Sample Lesson & Activity

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Lesson: A6.8 Arithmetic Sequences
Activity: A6.8 Arithmetic Sequence Cards

Because our materials reach thousands of students across several districts, we intentionally make them robust so they can be cut down and tailored to individual students. We never expect a Saga Fellow to cover everything within a lesson. We include an ample number of problems and tutoring strategies so Fellows can pick and choose what works best to meet their students' needs.

Activities are a way for students to dig into the concepts conceptually from a different point of view. Our activities are intentionally more open-ended than our problem sets, with many options for implementing to meet student needs.

Standards for Math Practice
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Objective A6.8: Identify and create arithmetic sequences.
F-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

A6.8 Fellow Implementation Notes & Key

Key Points

(1) A sequence is an ordered list of numbers that creates a pattern.
(2) An arithmetic sequence is created by adding the same value, called the common difference, to each term in order to determine the next term.

Lesson Overview

Students will learn that an arithmetic sequence is a list of numbers where a constant is being added to each term in order to get the next term. The constant that is added to each term in the pattern is called the common difference. Students will learn how to identify lists of numbers that are arithmetic sequences. They will also learn how to model both visual patterns and real life situations using the formula for arithmetic sequences, $a_n = a_1 + d(n - 1)$.

Potential misconceptions and errors:

(1) Students may not understand the meaning of the components in the formula, $a_n = a_1 + d(n - 1)$. a_n represents the n^{th} term in the sequence, a_1 represents the first term in the sequence, and d represents the common difference between subsequent terms. $(n - 1)$ represents the term number minus 1. Since a_1 (the first term) is already represented in the formula, we need to subtract out the first term so that we do not count it twice. Have students write the formula in their math notebook and label its components.
(2) Students may confuse the language they should use when discussing visual patterns vs. discussing arithmetic sequences. Arithmetic sequences have terms and term numbers while visual patterns have figures and figure numbers.

Key terms: arithmetic sequence, common difference, terms, patterns

Background Skills/Do Now

(See Lesson F1.13 Patterns)

1. Find the next three terms in each pattern.

a) 2, 4, 6, 8, …

10, 12, 14

b) 12, 9, 6, 3, …

0, -3, -6

(See Lesson A6.7 Model & Solve Linear Functions)

2. Write an equation to model the following data.

x	f(x)
1	5
2	9
3	13

$f(x) = 1 + 4x$

3. *Review From Yesterday*

4. *Spiral Review*

Opening Critical Thinking Task

Purpose of the opening task: Explore and introduce arithmetic sequences.

1. Compare the two patterns. What is similar between the two? What is different?

Pattern one: 2, 4, 6, 8, …

Pattern two: 12, 9, 6, 3,…

They are both changing by a steady rate. The first one is increasing by 2 each time and the second is decreasing by 3.

2. Find the next three terms in pattern one and pattern two. (*Hint:* You may have already done this in the Do Now!) Show or explain your reasoning.

Pattern one: 10, 12, 14

8 + 2 = 10, 10 + 2 = 12, 12 + 2 = 14

OR I added the two (the rate of change) to each prior term to get the next.

Pattern two: 0, -3, -6

3 – 3 = 0, 0 – 3 = -3, -3 – 3 = -6

OR I added -3 (or subtracted 3) to each prior term to get the next.

3. An <u>arithmetic sequence</u> is an ordered pattern of numbers where a constant is being added to each term to determine the next term. Are the patterns arithmetic sequences? How do you know?

Both pattern one and pattern two are arithmetic, because they are changing by the same amount each time.

- Why is pattern two still an arithmetic sequence even though it's decreasing? *This can be seen as adding a negative*

4. The constant being added to each term in an arithmetic sequence is called the <u>common difference</u>. What is the common difference of each arithmetic sequence? How did you find it? Why does it make sense to call it the common difference?

The common difference for Pattern one is 2 and the common difference for Pattern two is -3. The common difference is determined by finding the difference between any term and the previous: 4 – 2 = 6 – 4 = 8 – 6 = 2, and 9 – 12 = 6 – 9 = 3 – 6 = -3. It makes sense to call it the common difference because we find it by subtracting.

5. An arithmetic sequence can be represented by the formula: $a_n = a_1 + d(n - 1)$, where d is the common difference and a_1 is the first term. Write an equation for each pattern using this formula. What do you think n and a_n represent?

Pattern one: $a_n = 2 + 2(n - 1)$

Pattern two: $a_n = 12 + -3(n - 1)$

a_n represents the value of the term, n.

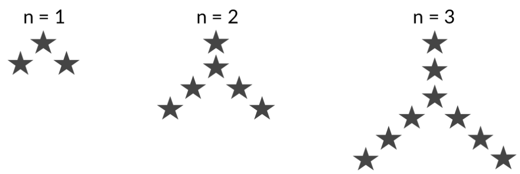
- How do we determine the 10^{th} term of the sequence for pattern one?
- What might be an advantage to using the formula to find terms for arithmetic sequences (instead of writing it out using the pattern)?

Example 1

Find the 110th term of the sequence 5, 9, 13, …. *$a_n = 5 + 4(n - 1)$* *$a_{110} = 5 + 4(110 - 1)$* *$a_{110} = 5 + 4(109)$* *$a_{110} = 5 + 436$* *$a_{110} = 441$*	*Purpose of Example:* Determine an explicit equation for arithmetic sequence and discuss characteristics of the explicit formula *Tutoring Strategies* Have students brainstorm possible strategies for solving. They may suggest writing out 110 terms of the sequence. Although this is a valid way of solving, it is inefficient. Students may also come up with other adapted or modified equations for the sequence. For example, a student may multiply 107 by 4 and then add 13. Push them to think about the most efficient model that can be used to represent any arithmetic sequence (the formula). • How do we know this sequence is arithmetic? *constantly increases by 4* • How could writing an equation to model the sequence help us? *Differentiation:* Have students start by finding the 10th term in the sequence manually. Then have them check that the equation they wrote gives the same result. Students were introduced to the formula $a_n = a_1 + d(n - 1)$ in the Critical Thinking Task. They can use this formula to find the 110th term. • What is the common difference? What is the first term? • Why is 1 being subtracted from n in the equation $a_n = a_1 + d(n - 1)$? *first term is already represented by a_1* Students may notice that they already modeled the sequence in this problem with a linear function for the Do Now. Compare the arithmetic sequence formula with the linear function for this sequence. • How is the equation from this problem different or similar to the equation from the Do Now? ○ What happens when we simplify the arithmetic sequence equation? *$a_n = 5 + 4n - 4 \rightarrow a_n = 4n + 1$* ○ How is the formula different or similar to a linear equation? • How are slope and the common difference related? *both rate of change* *Extend the example:* Have students discuss how they can find a missing middle term in an arithmetic sequence instead of a future term. • How would we find the missing term if we were given the arithmetic sequence: 5, ____, 13? *$\frac{13+5}{2} = 9$*

Example 2

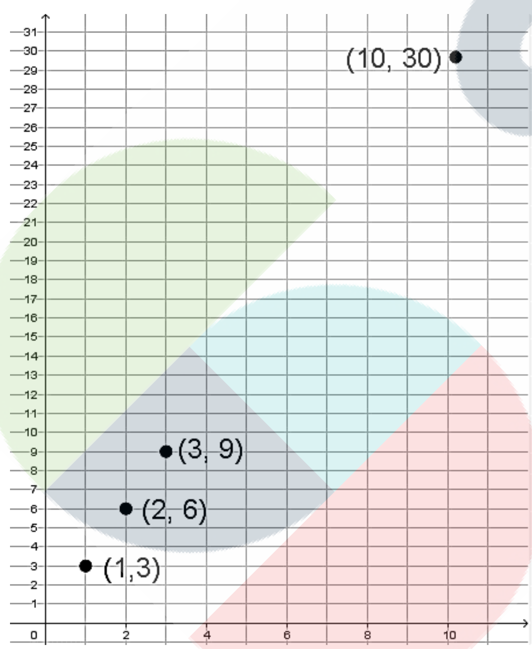
a) Find the number of stars in the 10^{th} figure of the pattern using an equation to model the number of stars.



Number of stars = 3 + 3(n – 1)
Number of stars = 3 + 3(10 – 1)
Number of stars = 3 + 3(9)
Number of stars = 3 + 27
Number of stars = 30 stars

b) Organize your data into the table below and graph.

Figure Number	Number of Stars
1	*3*
2	*6*
3	*9*
10	*30*



Purpose of Example: Derive formula from a visual pattern and graph the resulting sequence

Tutoring Strategies

a) Compare this example to the previous one. Have students describe how they see the pattern growing.

- How does the number of stars in each figure change?

Differentiation: Some students may find it helpful to use color to highlight how they see the star pattern changing from one figure to the next. This will help students visualize that the number of stars is increasing by 3 in each figure.

Discuss why a visual pattern is not considered an arithmetic sequence by itself. Have students discuss how we can use the formula for an arithmetic sequence to create an equation that models the visual pattern.

- Why is a visual pattern not an arithmetic sequence on its own? *not in the form of a list of numbers*
- Why can we use an arithmetic sequence to model the star pattern? *constant rate of change (adding 3 stars)*
- What arithmetic sequence models the number of stars? *3, 6, 9…*
  - What is the common difference?
  - What is the first term?
  - How can we write an equation for the arithmetic sequence?

Push students to consider the language we should use when discussing visual patterns vs. arithmetic sequences. Visual patterns have figures and figure numbers while arithmetic sequences have terms and term numbers.

b) Discuss how the graph relates to the visual pattern and arithmetic sequence.

- What information does the graph show in this case? *figure number and number of stars*
- What information does it lack? *what the pattern looks like*
- What type of graph would this make? *linear*
  - What does the slope represent? *rate of change*
  - Will all arithmetic sequences have a constant slope? *Yes, common difference is always constant rate of change.*
- Why isn't the y-intercept our first term? *y-intercept is at n = 0*

Practice Problems

Level 1

1. Fill in the blanks below using the word bank.

slope	term	linear	common difference
x-axis	pattern	y-axis	term number

An arithmetic sequence is an ordered _*pattern*_ in which a constant amount is added to each term to obtain the next term. It can be modeled by a graph, in which the _*common difference*_ is represented by the _*slope*_. On a graph, the _*x-axis*_ represents the _*term number*_, while the _*y-axis*_ represents the _*term*_.

Note to Fellows: The last sentence has two sections that can be interchanged, as long as the x-axis is linked to term number, and the y-axis is linked to the term.

2. Label each underlined part of the formula.

$$\underline{a_n} = \underline{a_1} + \underline{d}(\underline{n} - 1)$$

- a_n: Term
- a_1: First Term
- d: Common Difference
- n: Term Number

3. Find the common difference for each arithmetic sequence.

a) -34, -28, -22, -16, …
d = 6

b) 15, 10, 5, 0, …
d = -5

c) 0.9, 0.5, 0.1, -0.3, …
d = -0.4

d) 12, 22, 32, 42, 52, …
d = 10

e) 3, 8, 13, 18, …
d = 5

4. a) Complete the table for the sequence modeled below.

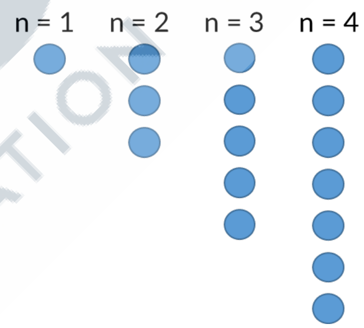


Figure Number	Number of Dots
1	*1*
2	*3*
3	*5*
4	*7*

b) Find the common difference.
d = 2

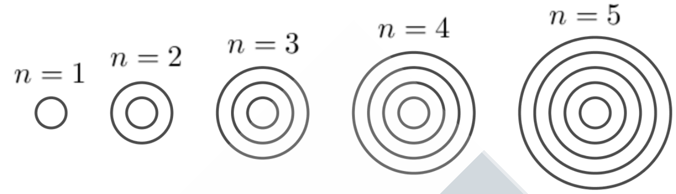
5. The following numbers are the start of an arithmetic sequence with common difference -20. Find the next four terms in the sequence. Explain how you found your response.

25, 5, …

-15, -35, -55, -75

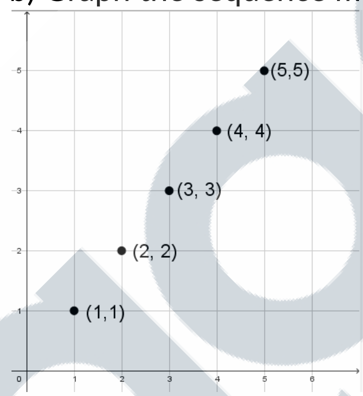
The common difference is -20, so if we continue to apply this common difference by subtracting 20 four times, we get -15, -35, -55, and -75.

6. a) The pattern below models a sequence. Find the common difference between the number of rings.



d = 1

b) Graph the sequence modeled by the pattern.



7. Find the common difference for the sequence represented by each table.

a)

n	1	2	3	4	5
a_n	16	15	14	13	12

d = -1

b)

n	1	2	3	4
a_n	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{9}{4}$

$d = \frac{2}{4} = \frac{1}{2}$

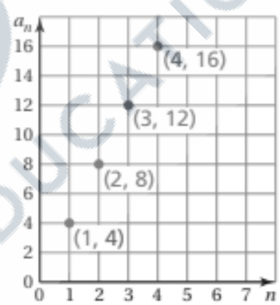
c)

n	1	2	3	4	5
a_n	2π	4π	6π	8π	10π

$d = 2\pi$

8. Determine the next three terms in the sequence modeled in each figure:

a)



$a_5 = 20$; $a_6 = 24$; $a_7 = 28$

b)

a_5 = 12 smileys
a_6 = 14 smileys
a_7 = 16 smileys

Image sources: Big Ideas Math

Level 2

9. Barrett says a common difference that's a negative means the sequence is decreasing. Is he correct? If so, explain. If not, give a counter example.

Yes, Barrett is correct. If the common difference is negative, then we must add a negative number to get from one term to the next term of the sequence, making the numbers decrease.

10. Find the missing terms in each arithmetic sequence.

a) <u>26</u>, 35, <u>44</u>, 53, …

common difference $= \frac{53-35}{2} = \frac{18}{2} = 9$

OR $35 + 2d = 53 \rightarrow d = 9$

b) 2, <u>6</u>, <u>10</u>, 14, ….

common difference $= \frac{14-2}{3} = \frac{12}{3} = 4$

OR $2 + 3d = 14 \rightarrow d = 4$

c) 3, <u>12</u>, 21, <u>30</u>, …

common difference $= \frac{21-3}{2} = \frac{18}{2} = 9$

OR $3 + 2d = 21 \rightarrow d = 9$

d) 3, <u>9</u>, <u>15</u>, 21, …

common difference $= \frac{21-3}{3} = \frac{18}{3} = 6$

OR $3 + 3d = 21 \rightarrow d = 6$

11. Find the 23rd term of the sequence:

19.5, 19.9, 20.3, 20.7, ….

$19.9 - 19.5 = 20.3 - 19.5 = 20.7 - 20.3 = 0.4$

Arithmetic sequence, common difference = 0.4

$a_n = 19.5 + 0.4(n - 1)$

$a_{23} = 19.5 + 0.4(23 - 1)$

$a_{23} = 28.3$

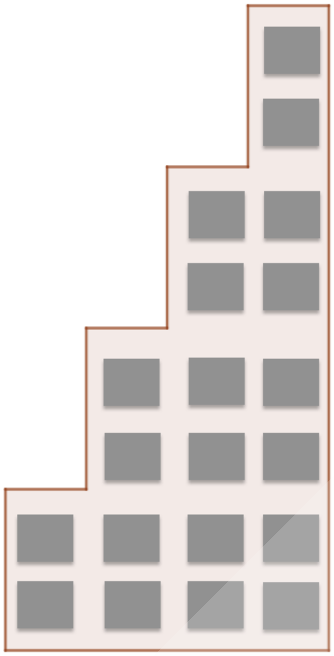
12. A molecule of water has two hydrogen atoms and one oxygen atom.

a) Write an equation to the model the total number of atoms, a, based on the number of molecules, n.

$a_n = 3 + 3(n - 1)$ *OR* $a_n = 3n$

b) Find the number of atoms in 35 water molecules.

$a_{35} = 3 + 3(35 - 1) = 3 + 3(34) = 105$

13. A side of an apartment building is shaped like a staircase. The windows are arranged in columns. How many windows are in the tallest column if the apartment building has 15 columns?  *Pattern: 2, 4, 6, 8, …* $a_n = 2 + 2(n - 1)$ OR $a_n = 2n$ $a_{15} = 2 + 2(15 - 1)$ $a_{15} = 2(15) = 30$ $a_{15} = 2 + 2(14)$ $a_{15} = 2 + 28$ $a_{15} = 30$	14. Estephanie just passed her road test and got her driver's license. She works at a movie theater part-time after school and wants to save up to buy her own car. On the first week, she saves $360 from her paycheck and then each week after, she saves $200 of her paycheck for her new car. How much money will Estephanie have saved after 12 weeks? $a_n = 360 + 200(n - 1)$ $a_{12} = 360 + 200(12 - 1)$ $a_{12} = 360 + 200(11)$ $a_{12} = 360 + 2200$ $a_{12} = 2560$ *Estephanie has $2,560 saved for her new car after 12 weeks.*

15. Trevor is volunteering with an organization called Rock the Vote to help members of his community register to vote. On the first day, Trevor's team helped 26 community members register and pledge to vote in the upcoming election. Every day after the first, they help 3 more people register and pledge to vote. How many community members will be registered to vote after 25 days?

$a_1 = 26$
$d = 3$
$a_n = 26 + 3(n - 1)$
$a_{25} = 26 + 3(25 - 1)$
$a_{25} = 26 + 3(24)$
$a_{25} = 26 + 72$
$a_{25} = 98$

98 community members will be registered to vote after 25 days.

16. Nadia is at her community's winter carnival. She is excited because this year there is a Ferris wheel, drop tower, and bumper cars. There is no fee to enter the carnival. However, it costs $3.00 for the first ride and then $1.50 for each additional ride. Nadia has $20 to spend at the carnival. If Nadia goes on 10 rides, will she have enough money left over to buy funnel cake that costs $2.50?

$a_1 = 3$
$d = 1.5$
$a_n = 3 + 1.5(n - 1)$
$a_{10} = 3 + 1.5(10 - 1)$
$a_{10} = 3 + 1.5(9)$
$a_{10} = 3 + 13.5$
$a_{10} = 16.5$
Nadia will spend $16.50 after 10 rides.
$20 – $16.5 = $3.50 so she will have enough money to buy the funnel cake.

17. Matthias solved the following problem as shown. Did he do it correctly? If so, explain his work. If not, explain and correct his error.

Fill in the missing term in the arithmetic sequence
2.2, _____, 6, ….

$$\frac{2.2 + 6}{2} = \frac{2.8}{2} = 1.4$$

The missing term is 1.4.

Matthias set up the problem correctly using the midpoint formula, because the missing term must be exactly in the middle of 2.2 and 6. However, he incorrectly added 2.2 and 6. His answer should have been:

$$\frac{2.2 + 6}{2} = \frac{8.2}{2} = 4.1$$

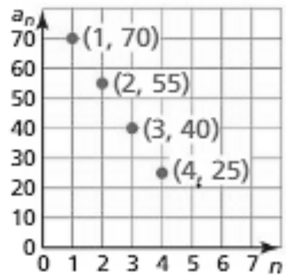
We can check our answer by finding the common difference between our three terms:
$4.1 - 2.2 = 1.9$
$6 - 4.1 = 1.9$ ✓

18. Diana says you only need two sequential terms of any arithmetic sequence to be able to write an equation for the sequence. Do you agree or disagree? Explain or show your reasoning.

Diana is right, given that we know that it is an arithmetic sequence. With two sequential terms, we can find the common difference. For example, for the sequence 4, 8, … $d = 4$ and $a_1 = 4$. However, if we are not told that this is an arithmetic sequence, then the pattern may be to multiply by 2, making the sequence 4, 8, 16, 32, … Without knowing the type of sequence, we could not write an equation to accurately model it.

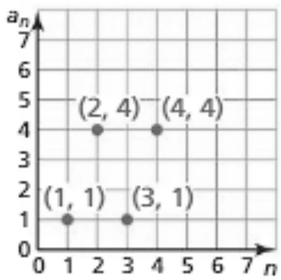
19. Determine if each models an arithmetic sequence and explain your reasoning. If it is, write the equation of the sequence the pattern is modeling.

a)



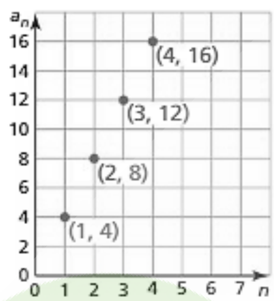
This models an arithmetic sequence. It decreases by 15 between each term.
$a_n = 70 + -15(n - 1)$

b)



This does not model an arithmetic sequence, because it does not have a single common difference.

c)



This models an arithmetic sequence. It increases by 4 between each term.
$a_n = 4 + 4(n - 1)$
OR
$a_n = 4n$

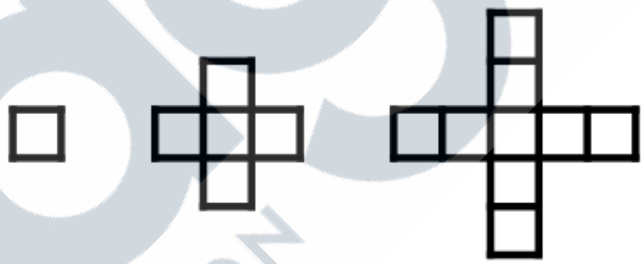
Image sources: Big Ideas Math

20. Determine if each pattern models an arithmetic sequence and explain your reasoning. If it is, write the equation of the sequence the pattern is modeling.

a)

This does not model an arithmetic sequence. Even though we can see that it grows in a predictable manner, there is no common difference. It grows 5 squares, then 7.

b)



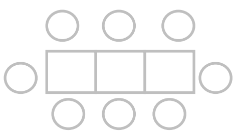
This models an arithmetic sequence. It grows by 4 both times. $a_n = 1 + 4(n - 1)$

Level 3	
21. Jeremiah is using the language learning app Duolingo to try and learn Spanish for fun. During the first week he learns 40 new phrases. After the first week, he learns 25 new phrases per week. How many weeks will it take Jeremiah to learn 3,015 phrases in Spanish? *$a_1 = 40$* *common difference = 25* *$a_n = 40 + 25(n - 1)$* *$3015 = 40 + 25n - 25$* *$3015 = 15 + 25n$* *$3000 = 25n$* *$120 = n$* *It will take Jeremiah 120 weeks to learn 3,015 phrases.*	22. Your school's auditorium has 20 rows total. The first row has 22 seats. The number of seats in each row increases by 6 as you move towards the back of the auditorium. For your graduation, your entire family wants to sit together in the last row. a) How many family members are you inviting to your graduation? *Answers will vary.* b) If 131 seats in the last row of the auditorium are already taken by other families, will your entire family be able to sit together in the last row? Justify your answer. *$a_1 = 22$* *common difference = 6* *$a_n = 22 + 6(n - 1)$* *$a_{20} = 22 + 6(20 - 1)$* *$a_{20} = 22 + 6(19)$* *$a_{20} = 136$* *$136 - 131 = 5$ seats are available in the last row* *Note to Fellows: If students invite 5 or less family members, then their entire family will be able to sit in the last row together.*

23. Gino is planning a big birthday party for his friend Angelina. He needs to rent enough tables for all of the guests. 1 square table seats 4 people. When 2 square tables are pushed together it seats 6 people.

a) How many people would 3 square tables that are pushed together seat? Show or explain your reasoning.

There are 3 people at each end table and 2 people seated in the middle. 3 tables pushed together seats 8 people.



b) How many tables would Gino need to push together in order to seat 105 guests at the birthday party?

Pattern: 4, 6, 8…

$a_n = 4 + 2(n - 1)$

$105 = 4 + 2n - 2$

$105 = 2 + 2n$

$103 = 2n$

$n = 51.5$

Gino needs to rent 52 tables to seat 105 guests

c) Will there be any extra seats? If so, how many?

$a_n = 4 + 2(n - 1)$

$a_{52} = 4 + 2(52 - 1)$

$a_{52} = 4 + 2(51)$

$a_{52} = 4 + 102$

$a_{52} = 106$

There will be 1 extra seat.

24. Yolanda is trying to become an influencer on Instagram. After the first month, Yolanda will have 30,000 followers. Every month after the first, she gains 1,250 more followers.

a) How many followers will Yolanda have after 6 months?

$a_1 = 30{,}000$

$d = 1{,}250$

$a_n = 30{,}000 + 1{,}250(n - 1)$

$a_6 = 30{,}000 + 1{,}250(6 - 1)$

$a_6 = 30{,}000 + 1{,}250(5)$

$a_6 = 30{,}000 + 6{,}250$

$a_6 = 36{,}250$

In 6 months, Yolanda will have 36,250 followers on Instagram.

b) Yolanda wants to be an influencer for Nike. She needs to have 100,000 Instagram followers before Nike will partner with her. How many months will it take Yolanda to have enough followers to become an influencer for Nike?

$a_n = 30{,}000 + 1{,}250(n - 1)$

$100{,}000 = 30{,}000 + 1{,}250(n - 1)$

$100{,}000 = 30{,}000 + 1{,}250n - 1{,}250$

$100{,}000 = 28{,}750 + 1{,}250n$

$71{,}250 = 1{,}250n$

$57 = n$

It will take Yolanda 57 months to have enough followers to become an influencer for Nike.

25. Create your own arithmetic sequence with a common difference of 4.5. List at least 3 terms. Explain how we know your sequence is an arithmetic sequence.

Answers will vary. Sample response:
1, 5.5, 10, …

26. Describe the 15th figure for each pattern.

a)

Number of Sides = 3 + 1(n – 1); $a_{15} = 3 + 1(15 - 1) = 17$
The 15th figure will be a 17th sided regular polygon.

b)

Diameters Drawn = 1 + 1(n – 1);
$a_{15} = 1 + 1(15 - 1) = 15$
The 15th figure will be a circle with 15 diameters drawn in.

27. This month Juanita listened to 12 new songs on Spotify because she wants to expand her music collection. She decides that she will listen to 15 new songs every month. How long will it take Juanita to listen to at least 500 new songs?

$a_1 = 12$
common difference = 15
$a_n = 12 + 15(n - 1)$
$500 = 12 + 15n - 15$
$500 = -3 + 15n$
$503 = 15n$
$33.5 = n$
It will take Juanita 34 months to listen to at least 500 new songs.

28. Approximately 1.6 million people in the United States do not have regular access to clean drinking water and sanitation necessities such as toilets and showers. However, it takes 1,800 gallons of water to produce 1 pound of beef.

a) Every McDonald's Big Mac uses one-half pound of beef. If one McDonald's restaurant in Washington DC sells 148 Big Macs today and then 175 Big Macs every day after, then how many gallons of water will the McDonald's use after 1 week?
1,800 gallons per pound of beef
1,800(0.5) = 900 gallons per half pound of beef
Day 1: 148(900) = 133,200 gallons
Common difference: 175(900) = 157,500 gallons
$a_n = 133{,}200 + 157{,}500(n - 1)$
$a_7 = 133{,}200 + 157{,}500(7 - 1)$
$a_7 = 1{,}078{,}200$
The McDonald's will use 1,078,200 gallons of water in 1 week.

b) Across the entire United States, an average of 1.5 million Big Macs are sold every day. How many gallons of water does the U.S. use for Big Macs every day?
1,500,000(0.5) = 750,000 pounds of beef per day
1,800(1,750,000) = 1,350,000,000
1.35 billion gallons of water is used for Big Macs every day in the United States.

c) Are you surprised by your answer to part b)? Do you view the water footprint left by the meat industry as a problem? Explain why or why not.
Answers will vary.

29. a) Kiara says that the range of an arithmetic sequence is always going to contain only positive and negative integers. She reasons that since the domain is composed of positive integers and the sequence is increasing or decreasing at a constant rate then the outputs will also be integer values. Is Kiara correct? Explain.

She is not correct. While the common difference is a constant rate, it is not necessarily an integer value. For example, for the sequence $a_n = 19.5 + 0.4(n - 1)$, the domain is positive integers, but the range has decimal values.

b) Under what conditions would the range be composed only of integers?

The outputs would be only integers if both the first term and the common difference are also integers.

30. Over the last ten years the number of inches of snow a town received formed an arithmetic sequence. 21 inches of snow fell 10 years ago and 19 inches fell 9 years ago.

a) What is the common difference?

Common difference = 19 – 21 = -2

b) Write a formula to model this sequence.

Answers will vary. Sample response:
If we let 10 years ago be represented by n = -10:
$a_n = 21 - 2(n + 10)$

Note to Fellow: Students may define n differently and create other valid formulas. This will affect the work shown in parts c) and d), but will not affect the final answers.

c) How many inches fell two years ago?

Two years ago is represented by n = -2:
$a_{-2} = 21 - 2(-2 + 10) = 21 - 2(8) = 21 - 16 = 5$
Two years ago, five inches of snow fell.

d) When will snow stop falling according to this model? What does this tell you about the validity of this model?

$0 = 21 - 2(n + 10)$
$0 = 21 - 2n - 20$
$0 = 1 - 2n$
$n = \frac{1}{2}$

According to the model, snow will stop falling in half a year in the town. This does not seem very likely though, so the model may not be accurate for this situation.

31. Korina donates the same amount of money each year to help protect the rainforest. At the end of the second year, she has donated enough money to protect 8 acres. At the end of the third year, she has donated enough to protect 12 acres. a) Does this represent an arithmetic sequence? How do you know? *Yes. We can use the context to conclude that since Korina donates the same amount each year, her money protects the same number of additional acres each year, and the sequence is arithmetic.* b) If Korina created a graph showing years vs number of acres saved, what would the graph look like? Explain how you know. *The graph would be linear, with a slope of 4 and a y-intercept of 0. This makes sense because the number of years and the number of acres saved are proportional. Before she started donating, she saved 0 acres, and each year she donates, she saves 4 more acres.* c) How many acres will her donations protect by the end of the tenth year? *Common difference = 12 – 8 = 4* $a_n = 0 + 4n$ OR $a_n = 4 + 4(n - 1)$ OR $a_n = 8 + 4(n - 2)$ OR $a_n = 12 + 4(n - 3)$ $a_{10} = 4(10) = 40$ *acres* *At the end of her tenth year, she will have saved 40 acres of rainforest.*	32. Use the sequences below to answer the questions. i. 2, 4, 6, 8… ii. -5, -10, -15, -20… iii. 7.5, 15, 22.5… a) Write an equation to model each. i. $a_n = 2 + 2(n - 1) = 2n$ ii. $a_n = -5 - 5(n - 1) = -5n$ iii. $a_n = 7.5 + 7.5(n - 1) = 7.5n$ b) Zhang correctly says we do not need to know about sequences to model this problem with an equation, although we could. What does he mean by this? *We have learned to model and write linear equations, so we could just write a linear equation to model each set of numbers without the use of the sequence formula.* c) What do you notice about the relationship between the first term and the common difference? *They are the same.* d) What would these sequences have in common if they were graphed? *They all go through the origin and are linear.* e) These arithmetic sequences can be modeled by proportional functions. How would you describe proportional functions? *Proportional functions have a constant rate of change, such that f(x) will change directly in proportion to the change in x. The equation can be modeled by f(x) = kx, where k is the rate of change. When graphed, they will cross through the origin and are linear.*

Challenge

33. A <u>geometric sequence</u> is an ordered pattern of numbers where a constant is being multiplied by each term to determine the next term. The constant being multiplied by each term in a geometric sequence is called the <u>common ratio</u>.

For example, for the sequence: 2, 4, 8, 16, …
The common ratio is 2.

Find the common ratio for each:
a) 3, 9, 27, 81…
common ratio: 3

b) 8, -16, 32…
common ratio: -2

c) 100, 50, 25…
common ratio: $\frac{1}{2}$ *or 0.5*

34. Find the missing terms in each geometric sequence. Determine the common ratio.

a) 4, _8_, 16, _32_, …
$4r^2 = 16 \rightarrow$ *common ratio = 2*

b) 3, _6_, 12, _24_, …
$3r^2 = 12 \rightarrow$ *common ratio = 2*

c) 1, _2_, _4_, 8, …
$1r^3 = 8 \rightarrow$ *common ratio = 2*

Ticket To Leave

1. a) Determine if the sequence is arithmetic. Explain.

5, 10, 15, 20, …

It is arithmetic because a constant (5) is begin added to each term.

b) Determine the 41^{st} term using an equation.
$a_{41} = 5 + 5(41 - 1) = 205$

2. Determine the next three terms of the sequence modeled in the table below.

n	1	2	3	4	5
a_n	25	23.5	22	20.5	19

$a_6 = 17.5$
$a_7 = 16$
$a_8 = 14.5$

b) What is the common difference? How do you know?
$d = -1.5$
$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = a_5 - a_4 = -1.5$

3. *Fellow's Choice*

4. *Fellow's Choice*

Arithmetic Sequence Cards Activity

This activity aligns with lesson A6.8 Arithmetic Sequences.

Cut out the card sets. Use as many or as few of the pieces (sequence, table, graph, description, equation, situation) as you would like, depending on the needs of your students.

Option 1: Mix all cards together and match the sets. This does not need to include all 6 cards in each set. You could choose to have students only match sequences to graphs. Or sequences to descriptions and situations. If you choose to only use the table, graph, equation, and situation, these cards could be applied to linear equations.

- Which was the hardest piece to match? Which were the easiest to pair up?
- Which situations (scenarios) could continue linearly forever in real life? Which wouldn't? When might they stop being a linear/arithmetic relationship?

Option 2: Draw one card from the set and come up with the other pieces. To do this as a team, have partners each pick a different card from a set and come up with the missing pieces. This can be done using teamwork, or done individually and checked as a team after. For example, Eveline picks a table card so she has to find the sequence, graph, description, equation, and situation that go with her table. From the same set of 6 cards, Malone chooses the graph card, so he has to find the sequence, table, description, equation, and situation that match his graph. After completing these, Eveline and Malone compare their answers.

- Which piece should always be identical? *sequence, table, graph, equation*
- Which pieces could vary student to student? *description could vary slightly, situation could vary drastically*

Option 3: Look for patterns in the graphs using only the sequence and graph cards.

- How does the first term in the sequence affect the graph?
  - How do you know if the graph starts above or below the x-axis?
- How does the common difference affect the graph?
  - How do you know if the graph slopes up or down?

Option 4: Relate the sequence to the equation by using only the sequence and equation cards.

- Based on the sequence, how do we know if the slope, m, in the equation y = mx + b should be positive or negative?
- Using the equation y = mx + b, is the constant b ever part of the sequence? Explain.
  - How do we determine b based on the sequence?
- In the equation y = mx + b, how does the sequence affect the value of m? How does it affect the value of b?

Option 5: Relate all cards by highlighting similar components. This can be done by first sorting the cards into groups of 6, or by using the sets of 6 without cutting the cards apart. Use two different colors to highlight/circle where we see the common difference and first term in each representation. If your students have already covered slope-intercept form of equations, you can also determine where you see the slope and the y-intercept in each representation, using a third color for y-intercept. (Students should determine slope is the same as the common difference, so it does not need a fourth color.)

- Where do we see the first term in each representation?
  - Where is it easiest to locate? hardest? Do you ever have to do a calculation to find it?
- Where do we see the common difference in each representation?
  - Where is it easiest to locate? hardest? Do you ever have to do a calculation to find it?

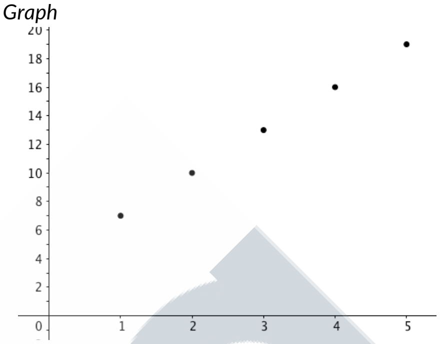
Sequence

7, 10, 13, 16, 19, …

Table

x	y
1	7
2	10
3	13
4	16
5	19

Graph



Description

Terms increase by 3, starting at 7

Equation

y = 3x + 4

Situation

Miriam earns $3 a day to feed her neighbor's cat. After the first day she had $7 in her wallet.

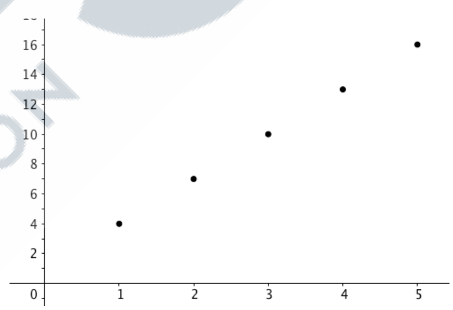
Sequence

4, 7, 10, 13, 16, …

Table

x	y
1	4
2	7
3	10
4	13
5	16

Graph



Description

Terms increase by 3, starting at 4

Equation

y = 3x + 1

Situation

Balzo earns $3 per week to mow the grass or rake leaves as a chore. After he collected his money on the first day, he had $4.

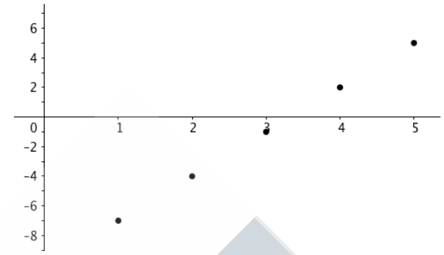
Sequence

-7, -4, -1, 2, 5, …

Table

x	y
1	-7
2	-4
3	-1
4	2
5	5

Graph



Description

Terms increase by 3, starting at -7

Equation

y = 3x – 10

Situation

Jordin earns $3 each day she walks a neighbor child to school. Before she started walking children to school, she was $10 in debt to her brother.

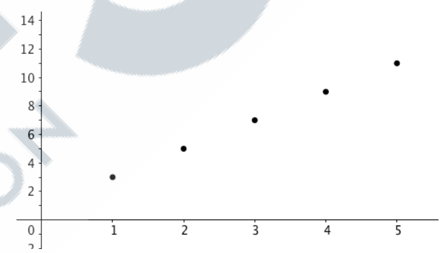
Sequence

3, 5, 7, 9, 11, …

Table

x	y
1	3
2	5
3	7
4	9
5	11

Graph



Description

Terms increase by 2, starting at 3

Equation

y = 2x + 1

Situation

Lupe likes to travel. Each year she travels to two more states. She started only having visited her state.

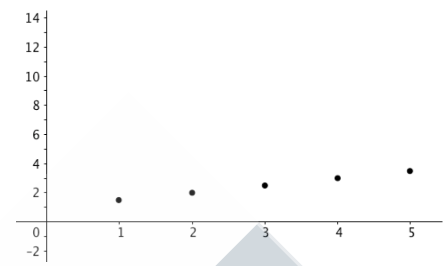
Sequence

$\frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$

Table

x	y
1	1.5
2	2
3	2.5
4	3
5	3.5

Graph



Description

Terms increase by $\frac{1}{2}$, starting at 1

Equation

$y = \frac{1}{2}x + 1$

Situation

Oscar likes botany so planted a one-inch-tall flower. Each week the flower grew half an inch, so it was 1.5 inches tall after 1 week.

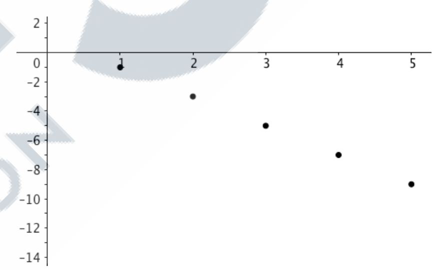
Sequence

-1, -3, -5, -7, -9, …

Table

x	y
1	-1
2	-3
3	-5
4	-7
5	-9

Graph



Description

Terms decrease by 2, starting at -1

Equation

$y = -2x + 1$

Situation

Zenobia's bank account charges her $2 a day for every day her account balance is under $10. On the first day she noticed she was charged, she was in debt $1.

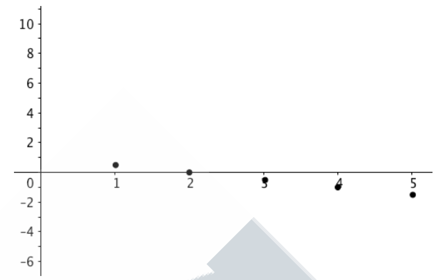
Sequence

$\frac{1}{2}, 0, -\frac{1}{2}, -1, -\frac{3}{2}, \ldots$

Table

x	y
1	0.5
2	0
3	-0.5
4	-1
5	-1.5

Graph



Description

Terms decrease by $\frac{1}{2}$, starting at $\frac{1}{2}$

Equation

$y = -\frac{1}{2}x + 1$

Situation

Bashir is playing a game where for every bad move he loses half a point. After his first bad move he has only half a point left, and he continues to make only bad moves.

Extension:
This GeoGebra widget allows students to explore the relationship between the first term, common difference, and the graph of a sequence: https://www.geogebra.org/m/kpAAYWc4